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Please use 'Highlights' in the file name and include 3 to 5 bullet points (maximum 85 characters, including spaces, per bullet point).

- Adding stochastic bias term to a process models improves reactor design reliability
- Stochastic bias description can be useful as an indicator of model structure errors
- Stochastic bias description does not guarantee reliability, it is not a panacea

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Accounting for erroneous model structures in biokinetic process models

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Abstract

In engineering practice, model-based design requires not only a good processbased model, but also a good description of stochastic disturbances and measurement errors to learn credible parameter values from observations. However, typical methods use Gaussian error models, which often cannot describe the complex temporal patterns of residuals. Consequently, this results in overconfidence in the identified parameters and, in turn, optimistic reactor designs. In this work, we assess the strengths and weaknesses of a method to statistically describe these patterns with autocorrelated error models. This method produces increased widths of the credible prediction intervals following the inclusion of the bias term, in turn leading to more conservative design choices. However, we also show that the augmented error model is not a universal tool, as its application cannot guarantee the desired reliability of the resulting wastewater reactor design.

Keywords: bias description; kinetic model; process design; wastewater treatment; uncertainty

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1 1. Introduction

In current environmental engineering practice, deterministic process-based 2 modeling is a common tool to better understand the functioning of complex 3 wastewater collection and treatment systems. The gold standard is to im-4 prove prediction performance of our models by fitting them to observations. 5 Consequently, the advent of ubiquitous sensing leads to an unintended yet 6 commonly observed situation where sensors reveal more details than mech-7 anistic models can capture. When this is the case, uncertainty estimates 8 obtained from statistical inference with mechanistic models are almost cer-9 tainly too narrow as the applied model structure is too restrictive relative 10 to the observed reality. A long-standing question is therefore whether risk-11 based design, based on uncertainty estimates from statistical inference with 12 mechanistic models, is actually feasible. In this work, we test one method 13 designed to address this issue to a case of WWTP design and discuss its 14 potential and limitations. 15

Accounting for model parameter uncertainty is crucial for risk-based decision-16 making, including infrastructure design and operations (e.g., Cagno et al., 17 2011; Scheidegger et al., 2013; Kabir et al., 2015; Scheidegger et al., 2015; 18 Jensen and Jerez, 2018). Conventional methods for uncertainty analysis are 19 based on a two-step approach, consisting of (a) quantification of input un-20 certainty, measurement uncertainty, and subsequent uncertainty of model 21 parameters followed by (b) propagation of the quantified uncertainty to the 22 system performance measure of interest (e.g., Van Griensven and Meixner, 23 2007; Sin et al., 2009; Guo and Murphy, 2012; Del Giudice et al., 2016). 24

²⁵ However, it has been demonstrated before how systematic deficiencies in
²⁶ model structure, next to input and measurement uncertainty, also lead to bi²⁷ ased model parameters and, consequently, incorrect design of infrastructural
²⁸ elements, such as biological reactor systems (Neumann and Gujer, 2008).
²⁹ Unfortunately, to our knowledge, no one has attempted to provide a method
³⁰ to solve this particular problem, i.e. to identify systematic discrepancies in
³¹ process-based models so to account for them during model-based design.

Recently, statisticians have been suggesting a promising approach to solve 32 this dilemma. The underlying idea is to not assume identically and indepen-33 dently distributed (i.i.d.) errors for mismatches between models and obser-34 vations (Liu and Zachara, 2001), but to explicitly account for mismatches by 35 adding a stochastic auto-correlated process to the i.i.d. measurement error 36 model (Craig et al., 2001; Kennedy and O'Hagan, 2001; Bayarri et al., 2007). 37 This is known as the bias description method and focuses on the modeling 38 of the symptoms of a mismatch between model structure and reality. While 39 this does not identify or tackle the root cause of these symptoms, it has 40 been proven to be a computationally efficient tool to increase the reliability 41 of model-based predictions compared to standard regression approaches in a 42 variety of systems from lakes to natural catchments to urban hydrology (Di-43 etzel and Reichert, 2012; Reichert and Schuwirth, 2012; Del Giudice et al., 44 2015). Therefore, we expect that the bias description method also improves 45 the reliability of predictions with structurally deficient wastewater treatment 46 models in view of risk-based design. Specifically, adding a stochastic mea-47 surement error term to a model given the same amount of experimental 48 measurements is expected to reduce the relative information-richness of the 49 experimental data and lead to larger credibility intervals of model parame-50 ters, wider prediction intervals and, by avoiding overconfident predictions, a 51 more trustworthy design. 52

Note that the bias description method can be regarded as a grey box or 53 hybrid modelling strategy. Indeed, the resulting model consists of a mecha-54 nistic model for the studied process (white box) and a stochastic model for 55 auto-correlated measurement errors (grey box). Other grey box approaches 56 may be baded on the inclusion of time-variant parameters (Reichert and 57 Mieleitner, 2009; Lin and Beck, 2012) or integration of non-parametric el-58 ements into a model structure that is mechanistic otherwise (Mašić et al., 59 2017). 60

In this contribution, we apply the bias description method to investigate the impact of model structure deficits for process design. We use Neumann and Gujer (2008) as a benchmark to evaluate the benefits and limitations of the bias description method for model-based design and refer to it as *the reference study*. While this reference study concerns a conceptually simple case, using it in this study highlights (a) that the apparent simplicity of this case is rather deceptive and (b) that challenges associated with model-reality mismatch are to be expected for both simple and complex systems.

⁶⁹ 2. Material and methods

70 2.1. Applied error models

In a vast majority of environmental modeling studies, the measurement 71 error is assumed to be i.i.d. For example, Hauduc et al. (2015) compares an 72 extensive list of model performance criteria for wastewater treatment mod-73 elling yet does not list any criterion which accounts for autocorrelated model 74 prediction errors. One approach considered in Cierkens et al. (2012), con-75 sists of downsampling time series to avoid the appearance of autocorrelation. 76 As explained in the same study, this leads to an inefficient use of the avail-77 able data and, more importantly, cannot account at all for model structure 78 deficits as a potential root cause of autocorrelated residuals. Ignoring the 79 presence of autocorrelated residuals was shown to lead to overconfidence in 80 the produced model and, subsequently, poor decision-making, as was shown 81 in the reference study. Most often, a Gaussian distribution is assumed for 82 the measurement errors. Such a model of measurement error cannot account 83 for systematic deviations between the assumed model and the observed mea-84 surements, i.e. bias. One way of accounting for bias is by adding terms, such 85 as a stochastic autocorrelated error bias term, to the measurement equation 86 (Craig et al., 2001; Kennedy and O'Hagan, 2001; Bayarri et al., 2007). In 87 this work, we describe the observable output time-series (i.e., measured con-88 centration, y_{0}) as a sum of a deterministic dynamic model output (y, the 89 modeled concentration), a classical Gaussian measurement error $(e(\psi))$, and 90 an auto-correlated error term $(b(\psi))$: 91

$$y_o(\theta, \gamma, \psi) = y(\theta) + \gamma + b(\psi) + e(\psi)$$
(1)

where θ and γ are parameters of the deterministic parts of the model and ψ are those of the stochastic parts (errors). The bias term $b(\psi)$ decribes an autocorrelated error $(b(\psi) \sim \mathcal{N}(0, \Sigma_b(\sigma_b, \tau)))$ and can be included to account

for time-dependent deviations between model and observations (Reichert and 95 Schuwirth, 2012). Note that this bias term represents a stochastic process, 96 thus describing aleatory uncertainty, although the deviations between model 97 and observations may actually be systematic, possibly even deterministic. 98 These deviations are expected to be systematic when they are caused by a 99 lack of knowledge about the true data-generating process. This lack of knowl-100 edge is typically characterized as a source of epistemic uncertainty rather than 101 aleatory uncertainty. 102

The bias term has two parameters, the standard deviation σ_b and the correlation length τ :

$$\Sigma_b(i,j) := \sigma_b^2 \cdot e^{-|t_i - t_j|^2/\tau}.$$
(2)

The random measurement error is temporally independent (e ~ $\mathcal{N}(0, \Sigma_e(\sigma_e))$) and is characterized by the parameter σ_e :

$$\Sigma_e(i,j) := \begin{cases} \sigma_e^2, & i=j\\ 0, & i\neq j \end{cases}.$$
(3)

Together, these error terms with parameters $\psi = \{\tau, \sigma_b, \sigma_e\}$ account for the fact that the deterministic model may not reproduce the modeled data set exactly. Note that the symbols σ_b and σ_e are chosen to convey the idea that they both describe the magnitude of variation of the a stochastic term in the measurement equation. The symbol for the correlation length, τ , is chosen to highlight the fact that it describes a time-scale.

¹¹³ The statistical formulation in Eq. 1 naturally leads to the likelihood ¹¹⁴ function $\mathcal{L}(y_o|\theta,\psi)$ which describes how likely the considered model with ¹¹⁵ parameters (θ,ψ) generated the recorded data, y_o . The likelihood of the ¹¹⁶ measurements conditional to the model parameters is:

$$\mathcal{L}(\mathbf{y}_o|\theta,\psi) = \frac{(2\pi)^{-\frac{n}{2}}}{\sqrt{\det(\Sigma)}} \exp\left(-\frac{1}{2} \left[\mathbf{y}_o - \mathbf{y}\right]^{\mathrm{T}} (\Sigma)^{-1} \left[\mathbf{y}_o - \mathbf{y}\right]\right)$$
(4)

where Σ is the variance-covariance matrix for the stochastic deviations between model and observations:

$$\Sigma := \Sigma_e + \Sigma_b,\tag{5}$$

with Σ_b and Σ_e defined in 2 and 3, and with *n* equal to the number of measurements. In order to interpret the results of parameter estimation, we define the parameters α and σ such that $\sigma_e^2 := (1 - \alpha) \sigma^2$ and $\sigma_b^2 := \alpha \sigma^2$. This means we can express the variance-covariance matrix above equivalently as:

$$\Sigma := \sigma^2 \cdot \left[(1 - \alpha) \ I_{n \times n} + \alpha \ K \right]$$
(6)

$$K(i,j) := e^{-|t_i - t_j|^2/\tau}$$
(7)

In this form, σ is a measure for the overall spread of the deviations be-124 tween the model and the measurements and α is a parameter that defines 125 the relative importance of the bias in the overall variance-covariance matrix. 126 Meaningful values for α are between 0 and 1, with $\alpha = 1$ leading to the omis-127 sion of the independent measurement noise ($\sigma_e = 0$) and $\alpha = 0$ expressing 128 that there is no bias ($\sigma_b = 0$). Note that setting $\alpha = 0$ reproduces the model 129 without a bias term. Put otherwise, the model with bias term includes the 130 model without bias term as special case. 131

The addition of a bias term accounts for underestimation of parameter uncertainty when a conventional yet unrealistic distribution for the model error is assumed (e.g., uncorrelated). This is expected to produce a wider predictive distribution, possibly leading to a better quantification of and a reduction of the risk of under-design or over-design. However, special attention must be given to the parameter estimation method as increased model flexibility can lead to unidentifiability (see e.g. Renard et al., 2010)

139 2.1.1. Model parameter estimation

We apply a Bayesian approach for two reasons. First, we favour a Bayesian framework as a way to make prior beliefs explicit. Second, without any form of prior, some of the parameters of the variance-covariance matrix Σ can be structurally unidentifiable (for definitions, see Dochain et al., 1995; Dochain and Vanrolleghem, 2001; Petersen et al., 2003). More specifically, when $\tau = 0$ the matrix K equals the identity matrix and likelihood $\mathcal{L}(y_o|\theta, \psi)$ becomes insensitive to the value of α . As a result, no unique value for α can be

identified under any circumstances as long as $\tau = 0$, i.e. α is structurally 147 unidentifiable. In the formulation with σ_e and σ_b , any increase of σ_e can 148 be compensated exactly by an equivalent decrease of σ_b when $\tau = 0$. For 149 small values of τ , e.g. close to the measurement interval or smaller, this is 150 expected to lead to lack of practical identifiability, even if structural identifi-151 ability could be guaranteed in principle. In early experiments with uniform 152 priors for τ , we observed that this can induce a lack of convergence and poor 153 mixing conditions for the applied sampling methods, similar to observations 154 described in Renard et al. (2010). Applying an informative prior solves this 155 identifiability problem and can therefore also be interpreted as a form of 156 regularization (e.g., Scales and Tenorio, 2001; Murphy, 2012; Hastie et al., 157 2015). 158

Bayesian calibration aims at characterizing the distribution described by the posterior likelihood $\mathcal{L}(\theta, \psi|\mathbf{y}_o) \propto \mathcal{L}(\mathbf{y}_o|\theta, \psi) \cdot \mathcal{L}(\theta, \psi)$, where the prior likelihood $\mathcal{L}(\theta, \psi)$ expresses the prior beliefs about the parameters. In this work, the posterior distribution is approximated with a sample of $\mathcal{L}(\theta, \psi|\mathbf{y}_o)$ drawn with a Markov Chain Monte Carlo (MCMC) sampler (see "Numerical implementation").

165 2.2. Biokinetic model parameter identification with batch experiments

To study the effects of model structure error and the utility of the bias description method, we execute simulations with the dynamic biokinetic model used in the reference study. Concretely, a series of batch experiments is simulated in which a substrate, with concentration s(t), is consumed by a cell culture with a fixed concentration. The conversion rate r(t) depends on the substrate by means of time-invariant Tessier kinetics so that one can write:

$$\frac{ds(t)}{dt} = -r(t) \tag{8}$$

$$s(t=0) = s_0$$
 (9)

$$r(t) = r_{max}^{Tessier} \cdot \left(1 - exp\left(-\frac{s(t)}{K_S^{Tessier}}\right)\right) \cdot x(t) \tag{10}$$

During the experiment, noisy measurements of the true substrate concentration, $y_o(t)$, are simulated by the following measurement error model, which is a zero-mean Gaussian noise term:

$$y_o(t) = s(t) + e(t)$$
 (11)

$$e(t) \sim N(0, \sigma_e) \tag{12}$$

Fixed parameters for each simulation are the same as in the reference 175 study: s_0 (initial substrate concentration, 5 g/m^3), r_{max} (maximum con-176 version rate, 1 $g/m^3 h$). The simulated time is T = 8 hours. The affinity 177 constant $(K_S^{Tessier})$ and measurement error standard deviation (σ_e) are varied 178 yet constant in every simulated experiment. $K_S^{Tessier}$ is varied from 0.1 g/m^3 179 to 1.5 g/m^3 in steps of 0.2 g/m^3 . This allows simulating a wide range of 180 process conditions, including both low and high values for $K_S^{Tessier}$ relative 181 to the initial substrate concentration. Two values for the simulated σ_e are 182 considered, as in the reference study. In the low-noise case, σ_e takes the value 183 0.01 g/m^3 . In the high-noise case, it takes the value 0.1 g/m^3 . The vector 184 θ equals $\left[s_0, r_{max}, K_S^{Tessier}\right]^{\mathrm{T}}$. The two simulated noisy time series obtained 185 with $K_S^{Tessier} = 0.7 \ g/m^3$ are shown in the supplementary information (Fig. 186 S.1 and Fig. S.2). 187

For each simulation experiment, parameter identification is executed with 188 four distinct model structures. The first model matches the above model 189 structure (Eq. 8-Eq. 12) exactly. This represents an idealized situation where 190 the structure of the calibrated model matches reality (ground truth) exactly. 191 The identified parameters are S_0 , $\mu_{max}^{Tessier}$, $K_S^{Tessier}$, and σ_e . A second model 192 is obtained by replacing the Tessier kinetics with the alternative and more 193 commonly used Monod kinetics. Practically, Eq. 10, is replaced with the 194 following equation: 195

$$r(t) = r_{max}^{Monod} \cdot \frac{s(t)}{K_s^{Monod} + s(t)}$$
(13)

The estimated parameters are now s_0 , μ_{max}^{Monod} , K_s^{Monod} , and σ_e with 196 $= [s_0, r_{max}, K_S^{Monod}]^{\mathrm{T}}$. This case represents the likely situation that a θ 197 modeling practitioner uses the common-place Monod model structure and 198 does not observe the bias that results. This is very likely in the high-noise 199 case (see reference study). Given this difficulty, the stochastic bias term de-200 scribed above is included to capture the systematic deviations between the 201 model predictions and measurements. To achieve this, the previously applied 202 measurement equation (Eq. 11) is replaced with the following equations: 203

$$y_o(t) = s(t) + b(t) + e(t)$$
 (14)

$$e(t) \sim N(0, \sigma_e) \tag{15}$$

$$\mathbf{b} \sim \mathcal{N}(0, \Sigma_b(\sigma_b, \tau)) \tag{16}$$

with the parameters τ defined as above and σ_b and σ_e reparametrized with 204 α and σ . This results in a third model, where a Tessier model is combined 205 with the statistical bias description and which requires specification of the 206 parameters s_0 , $\mu_{max}^{Tessier}$, $K_S^{Tessier}$, σ , α , and τ . As the Tessier model has 207 the same structure as the data-generating model, one can expect a good 208 model fit with α close to 0 and estimates of s_0 , $\mu_{max}^{Tessier}$, $K_S^{Tessier}$, and σ 209 that are close to ground truth values. Finally, the fourth model combines 210 the presumed Monod kinetics with the statistical bias description and the 211 identified parameters are s_0 , μ_{max}^{Monod} , K_s^{Monod} , σ , α , and τ . In this case we 212 can expect that the present model structure bias is accommodated by means 213 of the statistical bias description. If so, this should increase the width of the 214 prediction intervals and thereby improve the reliability of the model (Reichert 215 and Schuwirth, 2012). 216

217 2.3. Numerical implementation

The biokinetic model, parameter estimation, and uncertainty propaga-218 tion were implemented in Matlab (R2019a). The prior probabilities for the 219 parameters were set based on the authors' experience. They are all indepen-220 dent of each other. All priors are uniform, except for σ and τ . The prior 221 likelihood for σ is proportional to its inverse and is equivalent to the Jeffreys 222 prior conditional to fixed values for all other parameters (see Box and Tiao, 223 1973). The prior likelihood for τ is the sine function supported between 0 and 224 2 T. This prior equals zero at $\tau = 0$ and $\tau = 2$ T and one at $\tau = T$. This ex-225 presses the subjective belief that the autocorrelation length of the deviations 226 due to model structure error is expected to be similar to the duration of the 227 experiment. The priors are specified completely in Table 1. We first run an 228 adaptive MCMC algorithm (Vihola, 2012) to find a good guess for the maxi-229 mum a posteriori estimates and a good proposal variance-covariance matrix. 230 With these results, we execute a (non-adaptive) MCMC algorithm to obtain 231 20,000 samples from $\mathcal{L}(\theta, \psi|\mathbf{y}_{o})$. The first 10,000 samples are considered to 232 correspond to the burn-in phase of the sampler, during which effects of the 233 initial sample may still be apparent. These samples are therefore discarded, 234 as is common in practice (Gilks et al., 1996). 235

Parameter	Distribution	Lower bound	Upper bound	Unit
s_0	Uniform	0	$+\infty$	q/m^3
r_{max}	Uniform	0	$+\infty$	$g/m^3.h$
K_S	Uniform	0	$+\infty$	g/m^3
σ	Inverse $\left(\propto \frac{1}{\sigma}\right)$	e^{-12}	e^{+12}	g/m^3
α	Uniform	0	1	_
au	Sine $(\propto \sin(\frac{\pi}{2} \frac{\tau}{T}))$	0	2 T	h

2.4. Using information gained in parameter estimation for design 236

The obtained parameter estimates for each model are inspected by means 237 of visual inspection. In addition, we test the reliability of the calibrated 238 models for subsequent process design. To this end, the steady state substrate 239 concentration is computed for a CSTR with cells growing according to the 240 aforementioned ground truth Tessier kinetics. The dilution rate D is set at 241 $0.5 h^{-1}$, corresponding to a hydraulic residence time of 2 hours. The ground 242 truth steady state substrate concentration can be computed as follows: 243

$$s(t \to \infty)_{CSTR} = -K_s^{Tessier} \cdot \log\left(1 - \frac{D}{r_{max}}\right) \tag{17}$$

The above equation is also used to simulate the model-based concentra-244 tion estimate by replacing the true value of $K_s^{Tessier}$ with its estimates. To 245 do the same with the models exhibiting Monod kinetics, the steady state 246 concentration estimate is computed as follows: 247

$$s(t \to \infty)_{CSTR} = \frac{D \cdot K_s^{Monod}}{r_{max} - D}$$
(18)

The steady state concentrations are computed assuming that a perfectly 248 accurate value for r_{max} is available from a separate washout experiment (as in 249 the reference study). The affinity constant, K_s^{Monod} or $K_s^{Tessier}$, is therefore 250 the only parameter whose values are based on the batch experiment. In all 251 cases, the reliability of the predicted steady-state concentration in the CSTR 252 is assessed by visual inspection. Considering that the perfect information 253 about r_{max} is not accounted for during estimation of the affinity constant, we 254

also provide results in the supplementary information for a modified version of our method. In this case, we assume no washout experiment has been executed and use the sampled values for r_{max} in Eq. 17 and Eq. 18 instead of the assumed ground truth value.

259 2.5. Software availability

All data and numerical methods to reproduce our results are available as a stand-alone package for the Matlab (R2019b) platform. The version used for this article is added to the supplementary materials. The most recent version can be found on Gitlab (https://gitlab.com/krisvillez/biasdescription).

264 3. Results

265 3.1. Parameter estimation

The effect of bias and the use of the statistical bias description are demon-266 strated first by means of the marginal posterior parameter distributions ob-267 tained with data simulated with the Tessier model with $K_s^{Tessier} = 0.7 \ g/m^3$ 268 and $\sigma_e = 0.01 \ g/m^3$. In Fig. 1 the empirical cumulative density functions 269 are shown for the idealized case where the model structure is correct. The 270 results without bias description are shown with a dashed red line. One can 271 observe easily that the estimates of the deterministic part of the model are 272 fairly accurate. Indeed, the distributions are both narrow (all relative stan-273 dard deviations are below 2% in magnitude) and close to the true values 274 (all median estimates are within $\pm 1.5\%$ of the ground truth). This is not 275 surprising. In contrast, the median estimate of σ is about 35% smaller than 276 the simulated value while its precision remains small also (relative standard 277 deviation: 1.1%). 278

The red dashed lines in Fig. 2 show the empirical cumulative density 279 functions produced by using the Monod model structure while using the 280 conventional error model without bias description. This results in biased 281 estimates of the conversion rate (r_{max}^{Monod}) . These estimates do not reflect 282 the values used in the ground truth simulation, as discussed in the reference 283 study. For instance, the relative deviation median estimates are +19% for 284 r_{max}^{Monod} . This is explained as a consequence of using a different rate function 285 during estimation. Indeed, the parameter values for r_{max} (and K_S) are used 286 to compensate for the model structure error. The relative standard devia-287 tions for all parameters remain below 1.1% in magnitude, except for K_s^{Monod} 288 (2.7%).289



Figure 1: Batch experiment simulated with the Tessier model and $K_s = 0.7 \ g/m^3$ and $\sigma_e = 0.01 \ g/m^3$. Marginal posterior distributions for the calibrated Tessier model without bias description (dashed red line) and with bias description (full blue line). Vertical full lines indicate the true data-generating parameter values ($r_{max}, K_S, s_0, \sigma, \alpha$). Black dotted lines indicate the informative prior (α, τ).

This is different in the case where the Monod model structure is used 290 for estimation (Fig. 2). As expected, the posterior distribution for α is now 291 located to the right of its prior and indicates a large contribution of bias 292 to the prediction error. At the same time, the posterior for τ is to the 293 left of the prior, which means the autocorrelation length is shorter than 294 the experimental time length (8h). The residuals obtained with maximum 295 likelihood estimation of the Monod model now appear correlated as well (see 296 Fig. S.2 in the supplementary information). 297



Figure 2: Batch experiment simulated with the Monod model and $K_s = 0.7 \ g/m^3$ and $\sigma_e = 0.01 \ g/m^3$. Marginal posterior distributions for the calibrated Monod model without bias description (dashed red line) and with bias description (full blue line). Vertical full lines indicate the data-generating parameter values in the Tessier model used for simulation $(r_{max}, K_S, s_0, \sigma, \alpha)$. Black dotted lines indicate the informative prior (α, τ) .

298 3.2. Model-based prediction: use of parameter estimates for CSTR design

For the purpose of model-based design it is important to predict the steady state concentration properly for a given desired dilution rate. As indicated above, the steady state concentration is predicted at a dilution rate of $0.5 h^{-1}$ for both the ground truth as well as with the four constructed models.

The low-noise case is discussed first. The identified Monod model parameter sets are used to predict the steady-state concentration by means of Eq. 18. This is repeated for every simulated value of $K_S^{Tessier}$. The ratios of the predicted steady-state concentrations to the true steady-state concentra-

tion are visualized in Fig. 3. It is clear that using the Monod model leads to a significant bias in these predictions. This situation is however easy to identify by inspection of the posterior of α (as explained above). In addition, model structure error in the low-noise case is also identified easily by means of frequentist methods (cfr. reference study). We therefore assume that the modeler applies one of these tools and thereby successfully identifies the presence of bias.



Figure 3: Distributions of the ratio of predicted steady state concentrations to the ground truth concentration as a function of the affinity constant (K_S) – Low noise case ($\sigma_e = 0.01 \ g/m^3$). Red horizontal whiskers indicate the two-sided 99% credible intervals. Results are shown for the Monod model without bias description. As the ideal ratio of 1 falls outside the credibility intervals, we see that using a wrong model for reactor design will lead to a failure to achieve the desired pollutant removal capacity. This situation is easily detected in this low-noise case, also when using traditional methods.

As the high-noise case is more challenging for frequentist methods, we

discuss it in greater detail. Fig. 4 shows all results obtained for every assumed 316 structure for the deterministic model (Monod/Tessier) and for the stochastic 317 model parts (with and without bias). The top panel displays the results 318 obtained with the Tessier model. It is easy to see that the predictions are 319 unbiased as well as precise when no bias description is incorporated in the 320 error model (left-side bean plots), except when $K_S = 0.1 \ g/m^3$. We return 321 to the latter case below. If the stochastic bias term is added (right-side 322 bean plots), the distribution of the predictions becomes wider without any 323 meaningful shift of the average predictions $(K_S = 0.3 \ g/m^3$ and higher). 324 Consider that the process design is based on the shown 99% credible limits. 325 As adding bias increases the predicted upper limit, this will result in a larger 326 design volume to account for the perceived increase in uncertainty. As such, 327 accounting for uncertainty in the design would lead to a more conservative 328 design compared to the case without bias description. 329

The predictions obtained with a Monod model are shown in the bot-330 tom panel of Fig. 4. Depending on the ground truth value for $K_s^{Tessier}$, 331 rather severe under-prediction or over-prediction results when no bias term 332 is added (left-side bean plots). The predictive uncertainty (spread) is similar 333 to the case with the Tessier models without bias description. As a result, 334 the two-sided 99% credible intervals include the ground truth in only one of 335 the simulated cases $(K_s = 0.5 \ g/m^3)$, indicating the lack of reliability of the 336 model-based prediction intervals. The addition of a bias description term 337 abates this issue to some extent. Even though bias is still present, the ad-338 dition of a phenomenological bias term leads to increased widths of the the 339 credible intervals and a more conservative design. However, the ground truth 340 is included in the 99% credible intervals in two cases only $(K_s = 0.5 \ g/m^3)$, 341 $K_s = 0.7 g/m^3$). Thus, the bias description cannot ensure a reliable design 342 without further modification. Inspecting the posterior of α is more useful. 343 As one can see in Fig. 5, in all cases without model structure error (Tessier 344 model), the posterior of α is shifted towards the left of its prior, thus sug-345 gesting that the model structure is defined well. For the case with model 346 structure error (Monod model), the posterior is shifted to the right or re-347 mains relatively close to the prior, except for $K_S = 0.1 \ g/m^3$. One reason 348 is that this value is equal to the measurement error standard deviation. A 349 second likely factor is that the length of time during which the produced 350 experimental data are sensitive to the value of K_S is very short. Indeed, 351 the substrate concentration in the simulated experiment is between half and 352 twice the value for K_S for less than 3% of the duration of the experiment. 353



Figure 4: Distributions of the ratio of predicted steady state concentrations to the ground truth concentration as a function of the affinity constant (K_S) – High noise case ($\sigma_e = 0.1 \ g/m^3$). Red horizontal whiskers indicate the two-sided 99% credible intervals. Left side beans: without bias description; Right side beans: with bias description. Top: Tessier model - All credible intervals include the ideal ratio (equal to 1), except for the simulation with $K_S = 0.1 \ g/m^3$ without bias term. The uncertainty increases when a bias term is added to the model. Bottom: Monod model – Including the bias term in the model increases the reliability of the credible intervals. These intervals include the ideal ratio for two cases ($K_S = 0.5$ and $0.7 \ g/m^3$).

For $K_S = 0.3 \ g/m^3$ this already amounts to 8.6%. Since the model structure error primarily relates to the curvature of the conversion rate in this region, it follows that model structure error will always be difficult to detect when this time fraction is low.

In the supplementary information, we provide results obtained with the modified method. We omit the information obtained during the washout experiment during prediction. In this case, the uncertainty in the predictions



Figure 5: Distributions of the parameter α for both models with a bias term in all highnoise cases ($\sigma_e = 0.1 \ g/m^3$). When the Tessier model is selected (no model structure error), the posterior distribution of α is shifted to the left of the prior, thus suggesting the kinetic model structure is adequate. In contrast, the posterior of α is similar to or located at the right of the prior when the Monod model is used in all but one case ($K_S = 0.1 \ g/m^3$), thus providing a useful indication of model structure error.

is reduced significantly to the point that none of the 99% credible intervals include the ground truth (see Fig.S.3). This is explained by the fact that the estimates for r_{max} and K_S exhibit strong correlation (see supplementary information for details). However, since the modification relates to the prediction step only, one can still use the posteriors for α as a detection mechanism for bias.

367 4. Discussion

368 4.1. Summary and limitations of the experimental simulation study

Summary. The numerical results described above suggest that the inclusion 369 of an additive auto-correlated error process into a measurement error model 370 can improve the reliability of model-based designs. This is true even when 371 only a subset of the identified parameters are used during prediction (here we 372 only used the estimates for K_S and even when the experimental setting for 373 prediction (steady state) is different from the experimental conditions used 374 for model identification (batch experiment). In our case, the bias description 375 method improves the reliability in all cases. Despite this improvement, the 376 computed credible intervals include the ground truth value only in a lim-377 ited number of cases with model structure error, meaning that guaranteed 378 reliability cannot be obtained with the studied method. Thus, the inclusion 379 of a bias description term for the purpose of prediction can be advised as 380 a relatively fast and easy way to account for errors in the proposed model 381 structure, however only when one is unable to modify the model structure 382 itself. This is especially relevant in engineering applications where one is re-383 stricted to specific process representations (e.g., Monod kinetics) or software 384 with limited flexibility. While the bias description method improves the re-385 liability of the model predicitions only in a limited way, it is very useful as 386 a tool to detect the presence of bias during model identification, especially 387 when reformulated with the α parameter. 388

Limitations. In this study, a simple case was chosen deliberately for two reasons. First, this enabled an objective comparison of the bias descriptive method with the historical results in the reference study (Neumann and Gujer, 2008). Second, the apparent simplicity of the case also highlights the challenge of generating reliable predictions with mechanistic models, induced by the typical lack of flexibility of such models. The chosen scope also means that our study comes with some limitations, which are:

• The general applicability of the bias description method is not demonstrated. However, the bias description method could easily be adapted to more complex systems. One could incorporate a bias term to express correlation between multiple measurements, of the same or distinct variables measured in the same location or different locations. In this case, the covariance between two measurements, as expressed by Σ , would not only be a function of (a) the time difference $(t_i - t_j, \text{ see } (6))$,

as in our study, but also of (b) spatial distance in one or more dimensions and (c) effects of measurement error correlation between distinct
sensors measuring the same or distinct variables. This generalization
of the present model is likely most convenient when the bias error term
is modelled as spatio-temporal Gaussian process (e.g., De Cesare et al.,
2001; Gneiting, 2002; Stein, 2005).

• The methods applied in both the reference study and ours are based on 409 methods that account for aleatory uncertainty only. However, the lack 410 of knowledge about the model structure is typically epistemic in nature 411 and may therefore be difficult to account for in this way. Epistemic 412 uncertainty may however be reduced by using more flexible models 413 (Mašić et al., 2017) while increasing parametric uncertainty, which can 414 be handled as an aleatory source of uncertainty with currently available 415 methods. Still, the adoption of alternative frameworks for uncertainty 416 analysis (Parsons, 2001; Rao et al., 2008) may be suited to handle 417 epistemic uncertainty directly. In summary, the handling of epistemic 418 uncertainty deserves more attention. 419

420 4.2. General consequences for practical uncertainty and reliability analysis

Utility of the bias description method. In our opinion, the detection of sys-421 tematic deviations between the assumed model structure and the data-generating 422 process is the most useful feature of the bias description method. For this 423 reason, we recommend that a model is inspected for bias by adding a bias 424 term in the assumed model, specifying a prior for alpha concentrated around 425 a strictly positive value, as suggested here, and inspecting the posterior of α 426 whenever an inappropriate model structure is suspected. Reformulation of 427 the error model (bias + measurement error) with α , σ , and τ proved very 428 helpful as it enables interpreting α as an indicator for the relative impor-429 tance of model structure error. In cases where the posterior probability mass 430 is not shifted towards zero, relative to the prior, the modeler should suspect 431 the presence of bias. When this is detected, potential model improvements 432 may include the use of time-dependent parameters (Reichert and Mieleitner, 433 2009; Lin and Beck, 2012) and/or input errors (Del Giudice et al., 2016) or 434 a change in model structure (Del Giudice et al., 2015; Mašić et al., 2017). 435 While the method increases the reliability of the obtained steady-state pol-436 lutant concentration predictions, it is important to note that the observation 437 of this benefit depends strongly on the root cause of the observed bias. For 438

this reason, detection of bias should be followed by exploratory analysis of 439 the residuals and development of a better model structure (e.g., Reichert 440 and Mieleitner, 2009; Del Giudice et al., 2013). We do not recommend ex-441 ploiting the bias term for prediction without search for the underlying causes 442 for model structure deficits, especially considering that the ground truth is 443 rarely included in the produced credible intervals. Ultimately, the utility 444 of any approach depends on whether it can successfully describe the rele-445 vant sources of the deviations between model predictions and the measured 446 variables (Brynjarsdóttir and O'Hagan, 2014; Wani et al., 2019). 447

Parameter interpretation and transferability. The mechanistic interpretation 448 of identified values for the parameters in the deterministic part of the model 449 is nearly impossible when bias is present. Adding an auto-correlated additive 450 error term contributes to a better reliability of the model predictions but can-451 not provide a clearer interpretation of the parameter values or a direction to 452 a more appropriate model structure. Indeed, the parameter estimates remain 453 biased. Importantly, this is a likely scenario in wastewater engineering due to 454 the extremely simplified representation of biological processes during model 455 construction. Furthermore, obtaining proofs of a lack of bias is extremely 456 difficult to achieve so that a straightforward interpretation of parameter val-457 ues is unlikely, even when the model structure may be appropriate. However, 458 grey-box or hybrid models may offer intepretability and transparency at the 459 cost of computational efforts (see introduction above). 460

Data quality. The quality of the simulated measurements in the studied case 461 is fairly high relative to current experience in the wastewater sector. However, 462 sensor hardware has become increasingly robust in the last three decades 463 (Olsson, 2012) and there is no obvious reason why this trend should stop 464 now. It is therefore reasonable to expect that the presence of bias can be 465 detected easily in the future, either by statistical tests for auto-correlation 466 of the residuals, as in the reference study, or with descriptive methods, as in 467 this study. This will also facilitate the modification of the model structure 468 in accordance to the envisioned high-quality data. 469

470 4.3. Future work

Through this work, we identified several avenues of further research. These include:

- Develop and study methods for parameter estimation and parameter interpretation under presence of model structure error.
- Develop a systematic approach to the formulation of prior distributions,
 especially when flexibility is at odds with model structure or parameter
 identifiability.
- Evaluation of experimental design methods to improve the chances of detection of model structure errors.
- Adopt and evaluate methods to handle epistemic uncertainty in model based process design and operation.

482 5. Conclusions

In this paper, we investigated the challenge of structural model deficits in risk-based reactor design. This is a relevant problem, because digitalization will improve sensor resolution and spatial coverage of reactors, which will reveal mismatches (i.e, bias) in our common engineering models (which have been developed in the data-scarce past, often by grab sampling). Autocorrelated mathematical formulations have been suggested to improve the description of such biases.

490 In summary, our study shows that

- Adding auto-correlation terms in the measurement error model as a
 way to account for model structure deficits significantly improves the
 reliability of biokinetic models.
- Bias description enables accounting for predictive uncertainty during
 process design to a large degree. This does not produce a guaranteed
 reliability of the resulting design however. It is therefore not a bulletproof solution to the presence of model-reality mismatch.
- The studied bias description method is an adequate tool to identify the
 presence of model structure deficits in presence of noisy experimental
 data.

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508 6. Author contributions

Dario del Giudice: Methodology, Software, Data analysis, Writing - Review & Editing. Marc B. Neumann: Methodology, Software, Data analysis, Writing - Review & Editing. Jörg Rieckermann: Conceptualization, Methodology, Writing - Review & Editing. Kris Villez: Methodology, Software, Formal analysis, Data analysis, Visualization, Writing - Original Draft.

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Declaration of interests

 \boxtimes The authors declare that they have no known competing financial interests or personal relationships that 60 uld have appeared to influence the work reported in this paper.

□The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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